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WHAT DO BEGINNER STUDENTS FROM A BRAZILIAN LICENTIATE DEGREE IN SCIENCE COURSE REVEAL FACING A THEMATIZED PROBLEM IN LINEAR DIOPHANTINE EQUATIONS CONTEXT?

Wagner Marcelo Pommer[1]

Abstract: We consider fundamental to encourage and valorize themes belonging to the conjunction of Elementary Theory of Numbers and Algebra in the basic education, areas that present interesting and simple understanding contexts which allow developing problem solving strategies. This paper aimed to present the results of the application and analysis of a problem thematized in the linear Diophantine equations to beginner students from a Brazilian Licentiate degree in Science course. The methodology employed was based on the four phases of Didactic Engineering - preliminary analyzes, conception and priori analysis, experimentation and posteriori analysis - described in Artigue (1996; 2014; 2020). From the results, it was observed that all the students mobilized the trial-and-error as an initial tool to seek the entire solutions, by using arithmetic calculations. There were no manifestations of using algebraic writing as a basic support for solving the proposed problem, which represented an indication of the lack to understand the role of algebraic writing as a tool and potentiating strategy of mathematical problem solving.

Keywords: linear diophantine equation; discrete mathematics; strategies; problem; didactic engineering.

1 Trabalho atualmente na graduação e no mestrado acadêmico associado ao curso de Ciências-Licenciatura, da UNIFESP, campus Diadema-SP. Sou bacharel em Engenharia Mecânica pela Universidade Mackenzie (1983) e Física pela PUC/SP (1996). Tenho especialização em Matemática, pela Universidade São Judas Tadeu (1995), mestrado acadêmico em Educação Matemática (PUSP, 2008) e doutorado em Educação (FEUSP, 2012). Minhas pesquisas se situam na área da Educação Matemática principalmente envolvendo temas na confluência da Teoria dos Números e da Álgebra, situando temas como a Teoria Elementar dos Números e Números Irracionais. Na parte teórico-metodológica o enfoque envolve a Didática da Matemática Francesa, especialmente na Engenharia Didática de Brousseau e Artigue, nos Registros de Representação Semiótica de Duval, na Teoria do Antropológico de Chevallard, na Metacognição e nos Núcleos de Significação. Curriculo: http://lattes.cnpq.br/4262149292744127

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O QUE REVELAM ALUNOS INICIANTES DE UM CURSO BRASILEIRO DE LICENCIATURA EM CIÊNCIAS DIANTE DE UM PROBLEMA SITUADO NO CONTEXTO DAS EQUAÇÕES DIOFANTINAS LINEARES?

Resumo: Consideramos essencial incentivar e valorizar temas presentes na conjunção da Teoria Elementar dos Números e da Álgebra na escolaridade básica, áreas que apresentam contextos interessantes e de simples compreensão que permitem desenvolver estratégias de resolução de problemas. Este artigo teve como objetivo apresentar os resultados da aplicação e análise de um problema tematizado nas Equações Diofantinas Lineares a alunos iniciantes de um curso de Licenciatura em Ciências no Brasil. A metodologia empregada se baseou nas quatro fases da Engenharia Didática – análises preliminares, concepção e análise *a priori*, experimentação e análise *a posteriori* - fundamentada em Artigue (1996; 2014; 2020). A partir dos resultados, foi observado que todos os alunos mobilizaram a tentativa e erro como ferramenta inicial para buscar as soluções inteiras, por meio de cálculos aritméticos. Não houve manifestações do uso da escrita algébrica como suporte básico para a resolução do problema proposto, o que representou um indicativo da falta de compreensão do papel da escrita algébrica como ferramenta e estratégia potencializadora da resolução de problemas matemáticos.

Palavras-chave: equação diofantina linear; matemática discreta; estratégias; problema; engenharia didática.

1 INTRODUCTION

The feasibility of practical problems in mathematics teaching, valued and highlighted in the Brazilian National Curriculum Parameters (Brazil, 1998) and the current Common National Curriculum Base (Brazil, 2018), was already approached by the ancient Egyptians and Babylonians, through charades, conjectures and a variety of problems, which were incorporated into Western culture.

Among these, the so-called linear indetermination problems, an area of study belonging to the Elementary Number theory[2], are rarely addressed in Brazilian basic education. This area reveals several possibilities of exploration that:

> [...] have contributions to problem solving, understanding and development of other mathematical concepts, illustrating the beauty of Mathematics and understanding the human aspects of the historical development of numbers (Campbell; Zazkis, 2002, p. 2).

The Brazilian Elementary School Mathematics curriculum presents topics of the Elementary Number theory by addressing integer numbers on themes as basic properties and operations, decomposition into prime factors, Euclidean division,

2 The theory of Numbers refers to the set of natural and integer numbers and presents themes such as sequences, functions, equations, groups, standards, multiple, dividers, prime numbers, modular congruence, Diophantine equation, combinatorial analysis and theory of codes, and some of these issues are addressed in basic education (Pommer, 2008).

divisibility study, multiples and divisors, greatest common divisor, least common multiple, prime numbers and the fundamental theorem of Arithmetic, according to Pommer (2008).

After presenting these topics, school books in Brazil occasionally reuse these Elementary Mathematics topics throughout the Elementary and High School studies. Authors as Ferrari (2002) and Vilian *et al.* (2021) point out to the possibility of greater exploitation of themes belonging to the conjunction of the Elementary Number theory with Algebra, noting that such topics may favor problem solving in which the application of algorithms is not primordial. This opening would allow the development of skills such as interpreting and conjecturing, as well as encouraging the use of diversified strategies[3] and the search for heuristics[4].

In this sense, Vilian *et al.* (2021, p. 540) recalls the consideration of 'The World Economic Forum', occurred in 2016, which calls the attention to the skills that students:

> […] should have after completing their university studies. These are, for example, the ability to solve problems comprehensively, the ability to think critically, the ability to creatively use already acquired knowledge in various contexts and cognitive flexibility enabling insight into the problem to be solved from different perspectives.

Santos (2016) made a research involving students from a math degree course, and pointed out a gap between Algebra developed in the disciplines of that course and school Algebra. Thus, a stage of knowledge ressignification about Algebra to be taught by the Mathematics teachers in the final years of Elementary School and the initial stages of High School courses becomes a priority.

Given this picture, the purpose of this research was to present the results of the application and analysis of a problem thematized in the Linear Diophantine equations to begginer students from a Brazilian Licentiate degree in Science course.

2 THEORETICAL ASSUMPTIONS

An important aspect of the Number theory is the possibility of being worked in an articulated and complementary way with Algebra, as Pommer (2008) pointed out. These two fields are part of the current Math curriculum, exposed in Brazil (2018), as they are fields that have historically intertwined, and allow questions to be formulated whose complete solution requires integrated concept management.

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3 Ferrari (2002) points out the relevance of activities that favor the use of diversified strategies, such as the trial-and-error (the most basic), direct verification through numerical calculations and the use of properties and concepts, in order to open up ways to the mathematical objects make sense to the students.

4 According to Pozo (1998), heuristic procedures or strategies are plans and goals that students use in the search for solutions in problem situations. In opposition, algorithmic procedures are based on predetermined rules and operations, which enable the solution directly and in a specific way.

These considerations are in accordance with the Brazilian current Common National Curriculum Base (Brazil, 2018), a document that portrays the need to contemplate mathematical studies integrating numbers, operations and properties, in an articulated development of Arithmetic and Algebra fields.

Pozo (1988) considers that problem solving is a basic tool and implies the acquisition of different procedures and strategies to achieve a particular goal. In particular, Campell and Zazkis (2002) highlight the Diophantine equations[5] as an integrating theme connecting Arithmetic and Algebra, especially considering a context of problem situations.

Thus, we assumed that the use of linear Diophantine equations[6] can occur in a context of problem situations and games, in an approach to promoting the reuse of multiples and dividers concepts, as well as the possibility of exploiting different resolution strategies, which could enrich the learning of Algebra in basic education.

In addition to the factors outlined, the study of the linear Diophantine equations may favor the overcoming of bipolar tension between Discrete Mathematics and Continuum Mathematics. This pair refers to two fundamental actions of mathematics, namely: count and measure. In general sense, Brolezzi (1996) points out that the etymology of the word 'discrete' comes from latin term *discretus*, which denotes discriminating, separating or distinguishing. On the other hand, the term 'continuous' refers to the idea of something that is immediately joined with another element.

Moura (2005) outlines that the presentation in basic education textbooks has enhanced a dichotomy between these currents, with prevalence of discrete in the early grades of Elementary School (study involving natural numbers, integers and non-negative rational numbers). From the 8th grade of Elementary School the emphasis is on the mathematics of the continuum, and it becomes predominant in High School, through the study of real numbers.

Brolezzi (1996) highlights the necessity of a movement to bring them closer. Discrete mathematics and continuum mathematics are not two disrupted parts. On the contrary, between these two currents there are an elegant and important interaction, which could and should be explored in the Elementary Mathematics teaching at Elementary and High School.

In this way, problem solving can favor students some ways to express throughout written and oral form, allowing the manifestation of different strategies for the search of the challenges answers. The arguments used to define the solution and those, perhaps rejected, allow them to reveal the paths chosen in certain forms

5 Diophantine equation is defined as "[...] an algebraic equation with one or more unknowns and whole coefficients, for which whole solutions are sought. An equation of this type may have no solution, or have a finite or infinite number of solutions" (Courant; Robbins, p. 59, 2000).

6 The linear Diophantine equation consider here contain two variables and takes the form $ax+by=c$, where $x,y \in Z$ and a, b, c are integer parameters.

of resolutions. Thus, when facing such challenges, the student would manifest their cognitive abilities involved in the learning process.

3 THE HISTORICAL-EPISTEMOLOGICAL CONTEXT OF LINEAR DIOPHANTINE EQUATIONS

Situations involving linear Diophantine equations date back to some ancient civilizations texts. Problems involving applications of linear equations of two variables, such as $a\mathbf{x} + b\mathbf{y} = c$, had already been approached by Babylonians, as highlight Zerhusen, Rakes and Meece (2005).

Dias (2000) adds that problems involving Diophantine equations were found in Aryabhata, a mathematician-astronomers from the classical age of ancient Hindu mathematics (c. 476-550 a.C.). Also, Brahmagupta[7] (c. 598-668 a.C.), another Hindu mathematician, worked with this theme and proposed a global method to determine the general solutions of linear Diophantine equations. So, Brahmagupta proposed:

> [...] a general solution for linear Diophantine equation [of form] ax + by = c, where **a**, **b** and **c** are whole numbers. In this type of equation, to have whole solutions, the greatest common divisor of **a** and **b** must divide **c**; and Brahmagupt knew that if **a** and **b** were primes with each other, all solutions of the equation were given by x = p + m.b; y = q - m.a, where 'm' is an arbitrary integer number (Boyer, 1973, p. 161).

Boyer (1973) reveals that Brahmagupta provided all the whole solutions of linear Diophantine equation[8], while Diophanto from Alexandria (c. 201-284 a.C.) had only provided a particular solution. Struik (1992) adds that Diophanto only admitted positive solutions of indeterminate equations, while Brahmagupta admitted negative solutions of this type of equation, but this was something that was provided by the studies of predecessors' Hindu astronomers.

Ore (1998) points out that the problems of undetermined equations are found in other periods of Mathematics History after Diophanto and Brahmagupta. Some cases that illustrate this point refer to Bhaskara Akaria (1114-1185 a.C.), an ancient Arabian mathematician (in the book Lilavati Mahaviracarya), Leonardo from Pisa (in Liber Abaci, about 1200 a.C.), Flos (work of 1225 a.C.) and Rydolff (c. 1526 a.C.). Ore (1998) reveals that much of the problems described in the above mentioned authors were solved by various strategies. Later, Euler (1707-1783 a.C.), in his book *Algebra*, devoted several considerations about this subject.

7 Lintz (1999) states that the Hindu work is immersed within the arabian culture.

8 Boyer (1973) points out that Brahmagupta used some examples of Diophantus' indeterminate equations, giving evidence of Greek influence in India or the possibility of a common source from the Babylonians. Another feature common to both is the use of syncopated language.

The proposed problem situations thematized in Linear Diophantine equations allows establishing a bridge between the Number theory and Algebra, through the manifestation of various resolution strategies.

Still, the possibility of seeking whole solutions opposes the tendency of the current Mathematics teaching to privilege topics within real numbers domain. It is worth remembering that the resolution in the whole numbers context allows emerging its own characteristics, which highlight important concepts and properties of the field of Arithmetic, usually treated at the beginning of Elementary School, which are later gradually minimized in High School teaching situations.

4 THE METHODOLOGY AND THE *PRIORI* ANALYSIS

For this research we used as methodology the resources of Didactic Engineering, described in Artigue (1996; 2014; 2020). According to Artigue (2014), didactic engineering emerged in "[…] the early eighties and by the way Didactic Engineering has accompanied the development of didactical research, both in its fundamental and applied dimensions" (p. 468).

Dos Santos and Alves (2017) tell us that the Didactic Engineering methodology was proposed in order to study the teaching and learning processes around a given mathematical object. This fact began to occur in the early eighties, when Didactic Engineering became associated with the elaboration of artificial genesis, linked to didactic situations, according to Guy Brousseau's theory of Didactic Situations (1986), proposed to introduce or develop a given mathematical concept. Subsequently, Didactic Engineering expanded, and was used in other areas of knowledge.

Didactic Engineering (DE) emerged as a research and development methodology:

> […] based on classroom realizations in form of sequences of lessons, informed by theory and putting to the test theoretical ideas. At that time, what was predominant in the French didactical community was the theory of didactical situations that had emerged in the late sixties. This theory became thus the natural support of DE. Its systemic perspective, constructions and values shaped DE which progressively became the research methodology privileged within this community. In fact, it would be more adequate to say that theoretical constructions and DE jointly developed along the eighties (Artigue, 2014, p. 469).

Artigue (2020) highlights that, since its genesis, Didactic Engineering stood out for being a methodology that did not follow the traditional methodology in educational research at that time. So, Didactic Engineering had been planned to be developed in four steps: preliminary analyzes, conception and *priori* analysis, experimentation and *posteriori* analysis. So, Didactic Engineering did not obey:

> […] the validation paradigm based on the comparison of control and experimental groups. Its validation is internal and based on the comparison between the *a priori* and *a posteriori* analyses of the didactic situations

involved. This methodological choice can be easily understood considering the educational culture in which DE has emerged. In this culture […] research in mathematics education (didactics of mathematics) is seen as a scientific field of its own whose ambition is the study of the intentional dissemination of mathematical knowledge through didactical systems. What is to be understood is the functioning of such didactical systems, and associated didactical phenomena, which requires entering into the intimacy of their functioning. Validating the hypotheses engaged in the conception phase of a DE cannot be thus a matter of comparison between experimental and control groups (Artigue, 2014, p. 472).

The first phase, denominated as 'Preliminary Analyzes', according to Artigue (1996; **2020),** take into account the didactic knowledge already acquired, the epistemological study of the contents to be developed in the research and the analysis of students' conceptions, difficulties and obstacles that mark their evolution.

The second phase, called 'Conception and Priori Analysis' has a descriptive and predictive character. An essential point of support for this stage "[...] lies in the fine prior analysis of the students' conceptions, difficulties and stubborn errors, and engineering is designed to provoke, in a controlled manner, the evolution of conceptions" (Artigue, 1996, p. 202).

According to Machado (2002), at this stage the didactic variables are identified, which correspond as those for which the choices of values cause changes in problem-solving strategies. In this research, they corresponded to the numerical values intrinsic to the problem, chosen to stimulate in the students, in a controlled manner, the need to seek new strategies for resolution a given problem.

The third stage – the experimentation - consisted of the development and application of a problem situation to a group of ten higher education students, volunteers from a science degree course at a public university at Sao Paulo State, in an extra-class time.

Finally, in the *posteriori* analysis and validation stage occurred the confrontation of the data obtained and the priori analysis, which allowed the interpretation of the results, in a local generalization motto with internal validation.

In the sequence, we applied and analyzed a problem thematized in the linear Diophantine equations to ten beginner students, grouped in pairs[9], from a Brazilian Licentiate degree in Science course located at Sao Paulo Federal University, in Sao Paulo state, in the year of 2022.

9 By their own initiative, the ten science licentiate degree course students chose their partner to form the pairs and we named them as D_1, D_2, D_3, D_4 and D_5.

5 THE *PRIORI* ANALYSIS

This activity presented the following problem situation:

Using a weighting apparatus (showed in the figure at right), we put a 20 kg weight[10] in one side.	

(a) Do a table to indicate the balance possibilities of the weighting apparatus by placing some 2 kg and 3 kg weights on the other dish.

(b) Write all the ordered pairs (x, y) to balance 20 kg, obtained in the previous item, where 'x' indicates the number of weights of 2 kg and 'y' indicates that of 3 kg weights.

(C) Represent these ordered pairs in the Cartesian plane.

(D) Write an algebraic expression to represent the balance of this weight apparatus.

The didactic variables involved in the elaboration of this problem situation were:

- the use of a situation involving a thought experience, as the weight apparatus is not used as a real experiment, but an allusion or metaphor to correspond the mathematical meaning of the equal sign with something related to an everyday common situation.

- weights of 2 kg and 3 kg and a total weight of 20 kg, natural numbers and easy to handle in written or mental calculations;

- relationship between numerical data, which requires organization in the search for four solutions;

For the development of the 'weight apparatus problem' we predict some solution strategies, indicated by E_i (i=1,2,...), arranged in a descending order of occurrence probability. This is indicated as it follows:

(a) Do a table to indicate the balance possibilities of the weighting apparatus by placing some 2 kg and 3 kg weights on the other dish.

E_{1a}: The choice of relevant variables to denote a generic letter, for instance, 'x' as the number 2 kg weights and 'y' the number of 3 kg weights' and the use of mental or explicit calculations, through of the attempt and error strategy, to fill the table:

x: the number 2 kg weights	10	7	4	1
y: the number 3 kg weights	0	2	4	6

10 We chose to use the popular term 'weight' instead of 'mass', the latter being the most correct one from the point of view of science.

E_{2a}: The implicit problem equating for the solutions search and the table filling: The student performs calculations replacing values of 'x' and 'y', indicating arithmetic calculations for the search for entire solution.

2.10+3.0 = 20; 2.7+3.2 = 20; 2.4+3.4 = 20; 2.1+3.6 = 20.

E_{3a}: The explicit problem equating of the problem to search for solutions and filling the table: The student explicitly equates the problem as 2x+3y = 20 and uses it to search for integer solutions, by choosing some values as 'x' and finding the correspondent values of 'y'.

(b) Write all the ordered pairs (x, y) to balance 20 kg, obtained in the previous item, where 'x' indicates the number of weights of 2 kg and 'y' indicates that of 3 kg weights.

E_{1b}: The use of Cartesian coordinates - denoted by (x,y) - to indicate the four solutions: (10,0); (7, 2); (4.4) and (1.6).

E_{2b}: The use of Cartesian coordinates - denoted by (x, y) - to indicate the 4 solutions (10.0); (7, 2); (4.4) and (1,6), in an organized manner.

(c) Represent these ordered pairs in the Cartesian plane.

E_{1c}: The use of the Cartesian plane to indicate the points described in item (b), joining them with a straight line, denoting ignorance of the discrete origin of the variables.

E_{2c}: The use of the Cartesian plan to indicate the points described in item (B), not joining them with a line, denoting knowledge of the discreet origin of the variables;

(d) Write an algebraic expression to represent the balance of this weight apparatus.

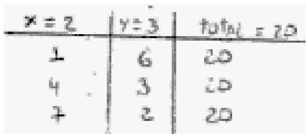
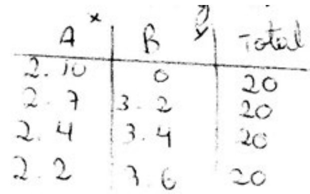
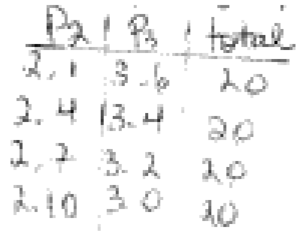
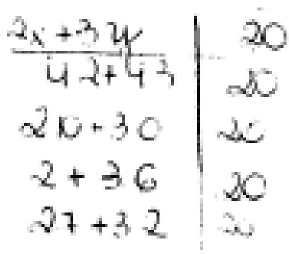
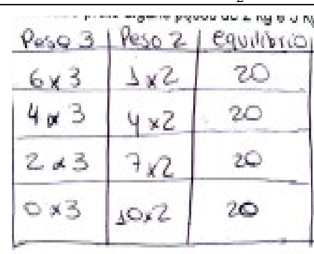
E_{1d}: Did not use the algebraic form to write down 2x+3y = 20.

E_{2d}: Correctly carried out the algebraic writing 2x+3y = 20.

6 DESCRIPTION AND *POSTERIORI* ANALYSIS OF THE 'WEIGHT APPARATUS PROBLEM'

For the activity, five pairs of students were formed. Each pair received a sheet with the statement of the activity and some space to register the solution process. The pairs of students interacted with the proposal, with the researcher playing the role of observer, mediator and institutionalizer. According to Brousseau (1986), the students were responsible for the necessary independent action to forward the resolution attempts, formulate possible solutions and subsequently share the results found.

Protocol 01 indicates the results obtained by the five pairs in item (a): "Do a table to indicate the balance possibilities of the weighting apparatus by placing some 2 kg and 3 kg weights on the other dish".

Protocol 01: Answers of the students to item (a) in the 'weight apparatus problem'.		
		
Answer from D_1.	Answer from D_2.	Answer from D_3.
		
Answer from D_4.	Answer from D_5.	

Source: The author, obtained from students' answers.

In this item, the pair D_1 organized the table in an ascending order, but one result was missing (10;0), as well as presenting a wrong result (4;3).

The other four groups set up the table through explicit calculations. It was notorious that the four pairs of students left the calculations that led them to indicate the answer, revealing the understanding of the equation as a means of justifying or organizing the results. Thus, in a certain way, they revealed the use of E_2 strategy, that is, implicitly used the equation of the problem to search for solutions and fill the table.

The pair D_5 organized the table indicating the values of the 3 kg weight and the 2 kg weight.

We noticed that the pair D_2 made a mistake on the calculations when presenting the last result of the table. The pairs D_2 and D_3 organized the table, a fact not observed in the pair D_4.

Protocol 02 presents the answers obtained in item (b): "Write all the ordered pairs (x, y) to balance 20 kg, obtained in the previous item, where 'x' indicates the number of weights of 2 kg and 'y' indicates that of 3 kg weights".

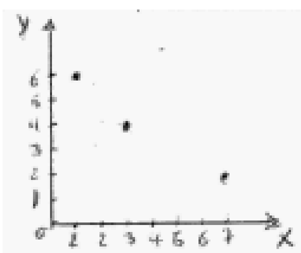
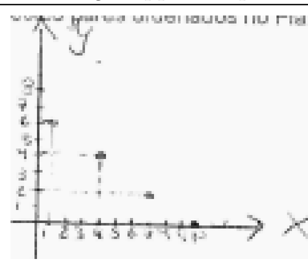
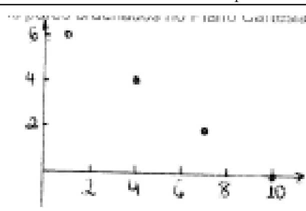
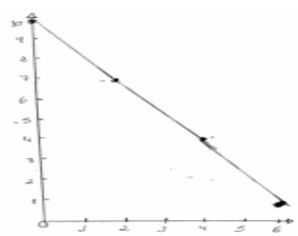
<table>
<tr><td colspan="3">Protocol 02: Answers of the students to item (b) in the ‘weight apparatus problem’.</td></tr>
<tr><td>(1;6); (4;3); (7,2)</td><td>(20,0); (14;6); (8,12); (4;16)</td><td>(1;6); (4;4); (7,2); (10;0)</td></tr>
<tr><td>Answer from D_1.</td><td>Answer from D_2.</td><td>Answer from D_3.</td></tr>
<tr><td colspan="2">(4;4); (10,0); (1;6); (7,2)</td><td>(6,1); (4,4); (2,7); (0,10)</td></tr>
<tr><td colspan="2">Answer from D_4.</td><td>Answer from D_5.</td></tr>
</table>

Source: The author, obtained from students’ answers.

We observed that the group D_2 made a mistake in writing the ordered pairs, indicating the partial results of the sums to assemble the table in item (a). Thus, we interpret that the pair D_2 did not understand the relationship between the ordered pair and the results obtained in item (a), that is, they did not understand the meaning of the variables in a problem situation.

The other four pairs hit the writing of the ordained pairs. The groups D_1 and D_3 ordered the results, revealing the use of E_2 strategy. On the other hand, the pairs D_4 and D_5 did not organize them, an indicative factor of the use of strategy E_1.

As for item (c), whose statement was “Represent these ordered pairs in the Cartesian plane”, the pairs D_1, D_3 and D_4 correctly represented the solutions in the Cartesian Plane, using E_2 strategy, as observed in protocol 03.

<table>
<tr><td colspan="2">Protocol 03: Answers of the students to item (c) in the ‘weight apparatus problem’.</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Answer from D_1.</td><td>Answer from D_3.</td></tr>
<tr><td></td><td></td></tr>
<tr><td>Answer from D_4.</td><td>Answer from D_5.</td></tr>
</table>

Source: The author, obtained from students’ answers.

The pair D_2 did not perform this item of the activity. Later, after the application of the activity, when questioned, the students revealed that they did not understand the issue.

The pair D_5 elaborated the graph correctly, but united the representative points of the solutions, not distinguishing the discrete variables from the continuous ones, indicative of using E_1 strategy.

For item (d), which requested "Write an algebraic expression to represent the balance of this weight apparatus", the pairs D_1, D_3 and D_4 correctly represented the solutions. The pair D_5 wrote down the equation correctly, according to the convention they adopted in item (a), when they placed in the double-entry table the amount of 3 kg and 2 kg weights, in that order.

Protocol 04: Answers of the students to item (d) in the 'weight apparatus problem'.			
2x + 3y = 20	2x+3y= 20	2x+3y=20	3x+2y=20
Answer from D_1.	Answer from D_3.	Answer from D_4.	Answer from D_5.

Source: The author, obtained from students' answers.

The pairs D_1, D_3, D_4 and D_5 made use of E_2 strategy, because they correctly performed the algebraic writing 2x + 3y = 20. The pair D_2 did not write down the algebraic equation (strategy E_1).

To provide a moment of synthesis, from the students' own manifestations, we made some questions, described below.

Question 01- Does this situation-problem represent a field in Mathematics? If yes, identify it.

Question 02- What do the data in the problem statements represent?

Regarding the 1st question 'Does this problem situation represent a field in Mathematics? If so, identify it.', the Table 01 summarizes the answers obtained.

Table 01: "Does this situation-problem represent a field in Mathematics? If yes, identify it".

Pair	**Answers**	**Students' comments**
D_1	Functions	As it envolves variables....
D_2	Equations	Did not justify.
D_3	Functions	Every value of 'x' has a 'y'.
D_4	Arithmetic progression	Because, in the table, the values of the weights are in an increasing and decreasing arithmetic progression.
D_5	Equations	Because it is necessary to discover values of the weights of 2 and 3 kg.

Source: The author, obtained from students' answers.

Through the students' responses, we concluded that the pairs delimited some possible Mathematics themes: equation, function and arithmetic progression[11].

The answers reveal vacant indications regarding the theme, as the Linear Diophantine equations (the characteristic mathematical object of this problem) represents a discreet function of first degree in the context of natural numbers.

The pair D_5 indicated the theme equations, by the presence of values of the weights of 2 and 3 kg. This denotes a lack of knowledge regarding the nature of the mathematics themes, in this case, the characteristics involving equations and functions.

Regarding the pairs D_1 and D_4, they recognized some aspects of this knowledge object, in general, without further deepening of the discreet nature of solutions and variables of the given problem.

The pair D_4 indicated some approximation to this issue, but they were guided by the numerical values, not explaining the idea of the variable element and the relationship between the two variables ('x' as the amount of 2kg and 'y' weights as the amount of 3kg weights).

Regarding the pair D_2, they indicated the theme equations, but did not have justified. It is noteworthy that this duo of students had serious gaps regarding the theme of functions, an essential item that belongs to High School curriculum in Brazil.

In the second question, 'What do the data in the problem statements represent?', Table 02 summarize the pairs answers.

Table 02: Answers to Question 02: What do the data in the problem statements represent?

Pair	Comments
D_1	They are the values that are part of the system and do not change, and allow a relationship between the values to be discovered.
D_2	Did not answer.
D_3	The data represent the not varying coefficients.
D_4	These are constants and from them you will develop the calculations.
D_5	The data represent the values to perform in the calculations.

Source: The author, obtained from students' answers.

The pairs D_1, D_3 and D_4 have some perception that data represent fixed numerical values. Still, the pairs D_1, D_3, D_4 and D_5 have an understanding of using

11 An arithmetic progression represents the study of a discrete function of 1st degree by an arithmetic and algebraic bias.

these values as a tool to operationalize numerical calculations, but have not showed understanding of what the variable term means.

Student responses indicated that there was an incipient discernment regarding the meaning of the data in the problem presented: the parameters. The answers make it expressive the difficulty to recognize and understand the values of the initial data - the weights of 2kg and 3 kg - as well as the total of 20 kg. In fact, these data represent the parameters, that is, the fixed values to the problem situation, which allow the amounts of 2kg and 3 kg weights to take different amounts.

7 FINAL CONSIDERATIONS

The approach of a problem involving a Linear Diophantine equation allowed mobilizing a diversity of resolution strategies, which enables locating the scope of relevance, validity and optimization of the possible choice of each one. Also, these questions enable, in a quite simple way, the manipulation of discreet quantities as solutions, discarding any continuous solutions, as well as the exploitation of situations with more than one or infinite solutions, fact not common in this education stage.

The strategies presented by the students may be situated in a path of searching the solutions of the given problem situation from trial-and-error method, a type of spontaneous students' strategy, as Ferrari (2002) points out. Historically, trial-and-error method was the main access route to resolve these problems until the 10th Century a. C.

Concerning the algebraic equation, most research subjects (80%) knew how to record the algebraic writing, but did not use it to seek the initial responses to solve and develop the problem presented. On the contrary, the manifestations indicated that the students still persist in making use of empirical calculations from Arithmetic to solve problems. This indicates a detriment of the use and understanding of algebraic writing role as a tool and potentiating strategy of mathematical problem solving.

We must remember that the goal of working with these types of problems is not to learn an algorithm that would allow getting the solutions of linear Diophantine equations, despising other strategies.

On the contrary, teaching should value procedures and knowledge as a tool to the greater teaching purpose, which should be to provide conditions for the student making sense of knowledge.

Here, we make an observation that Linear Diophantine equations represent one of the possible topics that allow the work with diversified strategies evolution, which includes the spontaneous participation of the students.

Also, the role of the teacher should be mapping the relevance, opening possibilities for the development of diverse competences, as highlighted by

Machado (2009), author who considers it possible to address any subject in classes, considering the appropriated scale, the scope and the teaching project.

Thus, the use of linear Diophantine equations theme, not as a curriculum component, but approached within contextualized problem situations, which were historically constituted and dialectically organized the own subject knowledge, which allows the development of essential competences in the basic cycle.

We add that linear Diophantine equations complement the usual curriculum themes, as they enable and revive their use, in a posture of appreciation of interdisciplinary, within the idea that knowledge is an artifact which is increasingly alive, since more connections are established to favor the meaning network of mathematical themes, within mathematics itself, according to Machado (2004).

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